

## Mathematical Basis For Descriptions of Vector Fields Using 'Curvilinear Squares' ?

The general problem of describing 2-D vector fields using "curvo-linear squares" and by implication 3-D vector fields using "curvo-linear cubes"

The teaching of magnetism in secondary school, seems to be where most people first come in contact with the description of vector-fields in terms of 'field-lines'; some like myself even being confused regarding the nature of these lines, ie whether they 'are the magnetic field' or as later realised merely 'describe the magnetic-field'.

There also seems to be a preference in engineering, more so than physics for describing vector fields in terms of "lines-of-force", 'flow-lines", stream-lines, field-lines...etc. In particular in electrical engineering the description and actual analysis of electric field into 'curvilinear-squares', defined by orthogonal sets of field-lines and equipotential lines.; each 'curvilinear-squares', such that " a circle touching all four 'sides' tangentially can be drawn or the bisector lines of each square are of equal length " according to different authors. Are these equivalent ?

see for example:"Curvilinear squares" in the teaching of electric fields (Authors:Henzl, Ctibor;Rozhon, Jan)  
:https://dspace5.zcu.cz/handle/11025/25838

Even in the mathematical engineering texts while field-lines generally may be defined, mostly they then fail to proceed to the complete mathematical description of the field in terms of a 'mesh system' of such field-lines and equipotential lines eg see appendix 1.1 Engineering Electro-magnetics (authors:William H. Hayt, Jr.John A. Buck) page 41-44

Which while approximating the vector direction at each point, fails to explain the way a complete description can also "map the field'. It seems to be implied that the density of intersection of such lines with any intersecting orthogonal equipotential surface will describe 'flux' density but does not make explicit the mathematical basis of this. Thus leaving the description of 2-D vector fields in terms of 'curvilinear-squares', without a mathematical basis, and by implication leaving the conception of a real 3-D vector field in terms of 'curvilinear-cubes' impossible. Leaving the student with the impression that such 'graphical' methods have no fundamental mathematical basis. see numerous references appendix 1/

Wikipedia has no article covering the subject: error message "The page "Curvilinear Squares" does not exist."  
<https://en.wikipedia.org/w/index.php?go=Go&search=Curvilinear+Squares&title=Special%3ASearch&ns0=1>  
and whilst 'field-lines' is given some mathematical attention see [https://en.wikipedia.org/wiki/Field\\_line](https://en.wikipedia.org/wiki/Field_line)  
it in no way leads to the idea of a field-map of a vector-field as suggested by the method of curvilinear-squares.

So here is the problem as I see it

1/Many physical vector fields imply a unique scalar constant the 'total flux'  
What is the mathematical requirement for this ?

2/Many physical vector fields imply another unique scalar constant the 'total potential'  
What is the mathematical requirement for this ?

3/ The ratio of these two scalars, often defining another important physical scalar constant for such fields,  
eg capacitance C for electrostatic fields, the resistance R for electrodynamic fields, the reluctance S for magnetic fields etc

4/It then seems possible to divide that 'total flux' into 'm' regions of equal 'flux', whose boundaries are surfaces (stream-surfaces ?), one of whose tangents at every point, describes the vector-field's direction at that point. Which tangent ?:

-For a 2-D field this surface is a stream-line, as a line whose tangents at every point describes the the vector-field's direction at that point.

- For a 3-D field it seems possible to further equally divide that region with a further set of 'm' orthogonal surfaces; ('stream-surfaces' ?) one of whose tangents at every point describes the the vector-field's direction at that point; with the total flux divided equally into  $m \times m$  'flux-tubes. (again which tangent ?:) Further in 3-D; the intersection of those orthogonal 'flow-surfaces' would seem to define ' $m^2$ ' stream-lines whose tangents at every point describe the vector-field's direction. What are the mathematical requirement for these ( ie 4/ down) ?

5/It also seems possible to divide that 'total potential' into 'n' equal 'regions', whose boundaries are surfaces, whose normals at every point describes the vector-field/s direction at that point and which are:

- in 2-D; 'n' lines orthogonal to the 'm' stream-lines and  
- in 3-D; are 'n' surfaces orthogonal to the both those ' $m \times m$ ' 'stream-surfaces' ?  
What are the mathematical requirement for these ( ie 5/ down) ?

6/ Thus we arrive by 'judicious selection' of 'm' and 'n' at a discrete description (map?) of some (all?)

- 2-D vector-fields using 'curvilinear squares' ie  $n \times m$  regions bounded on four sides, by 2 curved stream-lines and two curved equipotential lines. 'Curvilinear squares' with the geometric property: that a circle touching all four curves tangentially exists (true ?) This is essentially the graphical method of 'curvilinear squares'

Is this always possible ? " if so What should be the ratio of m to n ?

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7/ Thus we arrive 'by 'judicious selection' of 'm' and 'n' at a discrete description (map) of some (all?) 3-D vector fields using 'curvilinear cubes' ie 'm x m x n' curvilinear cubes' each bounded on six sides by curved surfaces, with the property that a sphere touching all six curves tangentially exists. This is essentially the hypothetical graphical method of 'curvilinear cubes' Is this always possible " if so What should be the ratio of m to n ?

What is the mathematical requirement for these ( ie 7/ down) ?

8/ Also we seem to have arrived at:

- one of a number of 'n' unique discrete descriptions (maps ? ) of some (all?) 2-D vector-fields and of one of a number 'n' unique discrete descriptions of it's 'n' stream-lines as boundaries of 'n' equal planar 'flux tubes'

- one of a number of 'n' unique discrete descriptions of some (all?) 3-D vector-fields and of one of a number 'n' unique discrete descriptions of it's 'n^2' stream-lines as vertices of 'n x n' equal square section 'flux tubes' .

Is this at variance with some traditional descriptions of vector-fields, using stream lines which seem to suggest that such descriptions can have arbitrary numbers of streamlines ?

Do stream-lines and equipotential-lines constitute an alternative reference frame for a vector field ?

appendix

eg 1.1 Engineering Electro-magnetics (authors:William H. Hayt, Jr.John A. Buck) page 41-44  
at :<https://www.pdfdrive.com/engineering-electromagnetics-e46174406.html>

eg-see P168 Electricity and Magnetism Oleg D. Jefimenko

eg: <https://www.pdfdrive.com/electricity-and-magnetism-an-introduction-to-the-theory-of-electric-and-magnetic-fields-e162096831.html>

-see page 58 Fundamentals of Electromagnetics 1: Internal Behavior of Lumped Elements  
David Voltmer [www.morganclaypool.com](http://www.morganclaypool.com)

eg :<https://www.pdfdrive.com/fundamentals-of-electromagnetics-1-internal-behavior-of-lumped-elements-e158457440.html>  
-see page 50-

Fields and Waves in Communication Electronics 3rd Ed [John Wiley and Sons]  
Simon Ramo, John R Whinnery, Theodore Van Duzier

eg:[https://www.readbookpage.com/access.php?](https://www.readbookpage.com/access.php?id=ABpRAAAAMAAJ&item=Fields%20and%20Waves%20in%20Communication%20Electronics)

[id=ABpRAAAAMAAJ&item=Fields%20and%20Waves%20in%20Communication%20Electronics](https://www.readbookpage.com/access.php?id=ABpRAAAAMAAJ&item=Fields%20and%20Waves%20in%20Communication%20Electronics)

-see page 156-159 Engineering Electromagnetics